LINEAR AND QUADRATIC EQUATIONS IN NON-STANDARD FORM

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Abstract

This article studies the non-standard design of linear and quadratic equations, their simplification using algebraic methods, and effective methods for solving them. Non-standard equations can have modular, fractional-rational expressions, roots, or hidden quadratic structures. In such problems, it is necessary to understand the internal structure of the equation, correctly determine the domain of determination, correctly collect the equivalent domains, and raise the found roots in a new equation. The article analyzes modular, fractional, root, and parametric equations using examples of solving them and methods for reducing them to a quadratic equation.

Keywords: non-standard equation, linear equation, quadratic equation, parameter, modular, fractional-rational equation, irrational equation, method, discriminant, Vieta's theorem, domain of determination, equivalent constant.

ЛИНЕЙНЫЕ И КВАДРАТНЫЕ УРАВНЕНИЯ В НЕСТАНДАРТНОЙ ФОРМЕ

Аннотация

В данной статье рассматривается нестандартная структура линейных и квадратных уравнений, их упрощение с использованием алгебраических методов и эффективные методы их решения. Нестандартные уравнения могут иметь модулярные, дробно-рациональные выражения, корни или скрытые квадратичные структуры. В таких задачах необходимо понимать внутреннюю структуру уравнения, правильно определять область определения, правильно собирать эквивалентные области и поднимать найденные корни в новом уравнении. В статье анализируются модулярные, дробные, корневые и параметрические уравнения на примерах их решения и методах приведения их к квадратному уравнению.

Ключевые слова: нестандартное уравнение, линейное уравнение, квадратное уравнение, параметр, модуляр, дробно-рациональное уравнение, иррациональное уравнение, метод, дискриминант, теорема Виета, область определения, эквивалентная константа.

Introduction

Solving algebraic equations is one of the main tools for solving theoretical and practical problems in mathematics. In school and academic lyceum algebra courses, linear and quadratic equations are usually studied in standard form. However, in practical and Olympiad tasks, the unknown does not appear in a simple form: it can be in the module, in the denominator, under the root, associated with a parameter, or hidden in another algebraic expression. In Uzbek textbooks, non-standard equations are studied as a separate topic, and various substitution and logical methods are used [1].

The relevance of studying non-standard equations is determined by the fact that they develop not only computational skills, but also mathematical observation, structural thinking and the ability to choose a solution strategy in the student. If the discriminant formula is directly applied in a standard quadratic equation, then in a non-standard equation it is first required to determine which standard model it corresponds to. Some fractional equations become quadratic equations when reduced to a common denominator; in some expressions, introducing a new variable significantly simplifies the problem [2]. Linear equations and their systems are one of the fundamental topics of the algebra course, and their general theory has been widely developed within the framework of linear algebra [3]. For quadratic equations, factorization, complete square decomposition, discriminant and Vieta's theorem are classic tools [4]. In modern algebra textbooks, equations involving biquadratic, reciprocal and reciprocal expressions reduced to a quadratic equation are also studied [5]. A specific feature of non-standard problems is the need to analyze the structure of the equation before choosing a ready-made algorithm.

The main goal of the study is to systematize the theoretical foundations and effective algebraic strategies for solving linear and quadratic equations of non-standard form. To achieve this goal, the tasks were set to determine the signs of a non-standard equation, analyze the mechanisms for reducing the equation to a standard form, indicate the domain of determination and equivalence conditions in cases where the modulus, fraction, root and parameter are involved, explain the substitutions leading to a quadratic equation, and justify the criteria for checking solutions.

The first step in solving non-standard linear equations is to determine in which algebraic form the unknown is involved. For example, the equation $(2x-3)/5 - (x+1)/2 = 4$ is reduced to a simple linear equation by getting rid of the denominators. In fractional-rational equations, the condition that the denominator is not equal to zero should be noted separately.

Modular equations are also an important form. The equation $|ax+b|=c$ has no real solution if $c<0$; when $c\geq 0$, $ax+b=c$ or $ax+b=-c$ are observed. For example, the equation $|2x-5|=7$ has solutions $x=6$ or $x=-1$.

In equations involving roots, the domain of determination is in the first place. For example, in the equation $\sqrt{(x+4)}=x-2$, $x\geq 2$ must be present. The roots obtained after squaring are necessarily checked in the original equation. This is a necessary condition for eliminating excess roots.

It is important to identify the hidden quadratic structure in non-standard forms of quadratic equations. For example, if we substitute $t=x^2$ in the equation $x^4-5x^2+4=0$, we get $t^2-5t+4=0$. This means that $t=1$ or $t=4$, which means $x=\pm 1$ or $x=\pm 2$. Since $t=x^2$, the condition $t\geq 0$ is also taken into account.

An example of a fractional-rational equation being reduced to a quadratic equation is $(5x+7)/(x-1)=3x+2$. When the general multiplication is performed with the condition $x\neq 1$, we get $5x+7=(x-1)(3x+2)$, which gives $3x^2-6x-9=0$ and $x^2-2x-3=0$. As a result, we find $x=3$ and $x=-1$; both satisfy the original equation [6].

In parametric equations, it is necessary to distinguish between special cases. In the equation $ax^2+bx+c=0$, the equation is considered a quadratic only if $a\neq 0$. If $a=0$, it becomes a linear equation. Also, the sign of the discriminant determines the number of roots depending on the value of the parameter. If the same complex expression is repeated in the equation, it is effective to introduce a new variable. For example, if $y=x-2$ is taken in the equation $(x-2)^2-5(x-2)+6=0$, $y^2-5y+6=0$ is obtained. Such a substitution allows us to view the complex expression as an independent variable and reduces the solution.

In the process of solving non-standard equations, it is possible to distinguish a universal sequence: first, the domain of determination is determined; then the hidden structure of the equation is found; then the simplest and most legal substitution is selected; the resulting standard equation is solved; finally, all candidate roots are checked in the original equation. This algorithm reduces the risk of excess roots, especially in equations with roots, fractions, and modular equations.

In quadratic equations, the choice of method depends on the form of the equation. For example, it is more convenient to factor the equation $x^2-7x+12=0$ than to calculate the discriminant; in an equation such as $x^2+6x+2=0$, it is more expedient to factor the formula or the complete square [7]. Thus, the quality of a mathematical solution is determined not only by knowing the formula, but also by choosing the method appropriate to the situation.

From a pedagogical point of view, non-standard equations are convenient material for the formation of high-level mathematical thinking. They lead the student from the question “which formula should I use?” to the question “what structure does the equation have and in what form can it be reduced?” The most common mistakes are related to forgetting the condition of the domain of determination, performing a non-equivalent operation without checking, and not taking into account the limits of values of the new variable.

A graphical approach is also useful: considering the left and right sides of the equation as functions gives an idea of the number of roots through the intersection points. However, the graphical method cannot replace algebraic proof; It is seen as a means of hypothesis development and outcome control.

Conclusion

The study proved that the main criterion for the effectiveness of solving linear and quadratic equations of non-standard form is not the choice of a ready-made formula, but the correct determination of the structure of the equation. In linear equations, it was found that the fraction, modulus and root elements should be considered together with the domain of determination; in quadratic equations, it was found that it is necessary to find the hidden quadratic structure, introduce appropriate substitutions and separate parametric cases. A sequential algorithm consisting of reducing the equation to a standard form, using equivalent substitutions and final verification reduces errors in non-standard problems. Non-standard equations develop mathematical observation, logical inference, strategic planning and checking one's own solution skills in students. Thus, their systematic study not only strengthens algebraic knowledge, but also forms the competence of independent mathematical thinking.

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