

THEME: REAL-LIFE APPLICATIONS OF ABSOLUTE VALUE IN MATHEMATICS

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Abstract. Absolute value, also called modulus, is a fundamental mathematical concept that represents the non-negative magnitude of a real number and its distance from zero on the number line. Although it is introduced as an elementary algebraic idea, absolute value has many practical applications in everyday life, science, engineering, economics, statistics, computing, navigation, measurement, and optimization. This article explains the mathematical properties of absolute value and demonstrates its real-life meaning through practical problems involving temperature, distance, financial change, measurement error, manufacturing tolerance, statistics, and coordinate geometry. Step-by-step examples show how absolute value converts signed differences into meaningful non-negative distances or magnitudes.

Keywords: absolute value, modulus, distance, number line, measurement error, tolerance, temperature, statistics, real-life mathematics.

1. Introduction

In mathematics, positive and negative numbers often describe direction or position relative to a reference point. However, many real-life questions are concerned with the size of a difference rather than its direction. Absolute value provides a simple

way to express this magnitude. It is therefore an important bridge between elementary algebra and practical quantitative reasoning.

The absolute value of a number x is written $|x|$. It represents the distance between x and zero on the real number line. Since distance cannot be negative, absolute value is always zero or positive.

2. Definition of Absolute Value

$$|x| = x, \text{ if } x \geq 0; \quad |x| = -x, \text{ if } x < 0$$

For example, $|7| = 7$ and $|-7| = 7$. Although 7 and -7 are located on opposite sides of zero, both are exactly seven units from zero.

Example 1. Find $|-12|$ and $|5|$.

Solution: The distance from -12 to zero is 12, so $|-12| = 12$. Since 5 is already positive, $|5| = 5$.

3. Absolute Value as Distance

One of the most useful interpretations of absolute value is distance. The distance between two real numbers a and b is the absolute value of their difference.

$$\text{Distance between } a \text{ and } b = |a - b|$$

Example 2. Two points on a road are located at kilometer markers 18 and 47. Find the distance between them.

Solution: Distance = $|47 - 18| = |29| = 29$ km.

Example 3. A diver is at -15 m relative to sea level and a drone is at $+35$ m. What is their vertical separation?

Solution: The separation is $|35 - (-15)| = |50| = 50$ m.

4. Temperature Applications

Temperature is a natural application because values may be above or below zero. If the question asks how much the temperature changed, the magnitude of the difference is often more important than the sign.

Example 4. The morning temperature is -6°C and the afternoon temperature is 8°C . Find the temperature difference.

Solution: $|8 - (-6)| = |14| = 14^{\circ}\text{C}$.

Example 5. Two cities have temperatures of -12°C and -3°C . How far apart are their temperatures?

Solution: $|-12 - (-3)| = |-9| = 9^{\circ}\text{C}$.

5. Financial Applications

Positive and negative numbers are commonly used to represent profit and loss, deposits and withdrawals, or increases and decreases. Absolute value can describe the magnitude of a financial change without focusing on its direction.

Example 6. A company records a change of $-\$2,500$ in monthly profit. What is the magnitude of the change?

Solution: $|-2500| = 2500$. The magnitude of the change is $\$2,500$.

The sign remains important when determining whether a change is a gain or a loss. Absolute value should be used only when the question asks for the size or magnitude of the change.

6. Measurement Error

Measurements rarely equal the exact theoretical value. Absolute error is defined as the absolute difference between a measured value and the accepted or true value.

$$\text{Absolute error} = |\text{Measured value} - \text{True value}|$$

Example 7. A rod should be 50.00 cm long, but its measured length is 49.94 cm. Find the absolute error.

Solution: $|49.94 - 50.00| = |-0.06| = 0.06$ cm.

7. Manufacturing Tolerance

Engineering and manufacturing frequently specify a target dimension together with an allowed tolerance. If the target is T and the measured value is x , a tolerance d can be written as an absolute-value inequality.

$$|x - T| \leq d$$

Example 8. A machine part should have a diameter of 20 mm with a tolerance of 0.05 mm. Express the acceptable range.

Solution: $|x - 20| \leq 0.05$. Therefore $19.95 \leq x \leq 20.05$ mm.

8. Navigation and Coordinate Geometry

On a straight east-west or north-south route, signed coordinates can represent positions relative to a reference location. The distance between two positions is obtained by taking the absolute difference.

Example 9. A delivery vehicle is at position $x = -8$ km and a warehouse is at $x = 13$ km. Find the distance between them.

Solution: $|13 - (-8)| = 21$ km.

In two-dimensional geometry, absolute values also appear in formulas for distances measured along coordinate directions and in Manhattan distance, which sums horizontal and vertical absolute differences.

$$\text{Manhattan distance} = |x_2 - x_1| + |y_2 - y_1|$$

9. Statistics and Data Analysis

Absolute value is useful when measuring how far observations are from a central value. The absolute deviation of a data value x from a mean or median m is $|x - m|$. Mean absolute deviation summarizes the average size of deviations without allowing positive and negative differences to cancel each other.

$$\text{Mean Absolute Deviation} = (\sum |x_i - \bar{x}|) / n$$

Example 10. The values are 4, 6, and 8. Find their mean absolute deviation.

Solution: The mean is 6. The absolute deviations are $|4-6|=2$, $|6-6|=0$, and $|8-6|=2$. Their average is $(2+0+2)/3 = 4/3 \approx 1.33$.

10. Absolute Value in Physics

Physical quantities such as displacement and velocity may carry signs to represent direction. Sometimes the magnitude alone is required. For one-dimensional motion, the magnitude of displacement can be written as an absolute value. However, students should distinguish distance from displacement: total distance traveled is not always equal to the absolute value of net displacement.

Example 11. An object moves from position -4 m to position 9 m. Find the magnitude of its displacement.

Solution: Displacement $= 9 - (-4) = 13$ m, and its magnitude is $|13| = 13$ m.

11. Absolute Value Equations

An equation such as $|x - a| = d$ describes all points located exactly d units from a on the number line. If $d > 0$, there are normally two solutions.

$$|x - a| = d \Rightarrow x = a + d \text{ or } x = a - d$$

Example 12. Solve $|x - 5| = 3$.

Solution: The number x is 3 units from 5. Therefore $x = 5 + 3 = 8$ or $x = 5 - 3 = 2$.

12. Absolute Value Inequalities

Absolute-value inequalities are useful for expressing acceptable ranges, error limits, and tolerances.

$$|x - a| < d \Rightarrow a - d < x < a + d$$

$$|x - a| > d \Rightarrow x < a - d \text{ or } x > a + d$$

Example 13. A room temperature should remain within 2°C of 22°C . Express the condition and determine the acceptable range.

Solution: The condition is $|T - 22| \leq 2$. Therefore $20^{\circ}\text{C} \leq T \leq 24^{\circ}\text{C}$.

13. Computing and Optimization

Absolute differences appear in computer algorithms that compare numerical values, evaluate prediction errors, process images, and optimize routes. In machine learning, the mean absolute error measures the average magnitude of prediction errors.

$$\text{MAE} = (1/n) \sum |y_i - \hat{y}_i|$$

Absolute-value-based loss functions are useful because they penalize errors according to their magnitude while treating positive and negative prediction errors symmetrically.

14. Summary of Real-Life Applications

Application	Absolute-Value Model	Meaning
Distance on a line	$ a - b $	Distance between two positions
Temperature	$ T_1 - T_2 $	Magnitude of temperature difference
Measurement error	$ \text{Measured} - \text{True} $	Size of measurement error
Manufacturing	$ x - T \leq d$	Acceptable tolerance
Statistics	$ x - \text{center} $	Magnitude of deviation

Finance	$ \text{change} $	Magnitude of gain/loss change
Computing	$ \text{actual} - \text{predicted} $	Prediction or numerical error

15. A Step-by-Step Problem-Solving Strategy

First, identify the reference value or two quantities being compared. Second, determine whether the problem asks for direction or only magnitude. Third, represent the difference algebraically. Fourth, apply absolute value when a non-negative distance or magnitude is required. Finally, interpret the numerical answer together with the correct unit.

Students should not automatically remove negative signs from every calculation. Absolute value has a specific meaning. It is appropriate when the magnitude of a signed quantity or the distance between values is required.

16. Educational Importance

Real-life applications help students understand that absolute value is more than a symbolic rule. Number-line distance provides a visual foundation, while temperature, finance, measurement, and tolerance problems show why the same concept is useful outside the classroom. Connecting algebraic expressions to practical contexts strengthens mathematical modeling and problem-solving skills.

17. Conclusion

Absolute value is a simple but powerful mathematical concept that expresses non-negative magnitude and distance. Its applications include temperature differences, geographic and coordinate distances, financial changes, measurement errors, engineering tolerances, statistical deviations, physical quantities, and computational error measures. The formula $|a - b|$ provides a universal way to measure separation on the real number line, while inequalities such as $|x - T| \leq d$ efficiently describe acceptable ranges. Understanding these real-life interpretations helps students apply

algebra meaningfully and prepares them for more advanced mathematics, science, engineering, statistics, and computer science.

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