

THEME: THE ORIGIN AND HISTORICAL DEVELOPMENT OF GEOMETRY

TOSHKENT DAVLAT IQTISODIYOT UNIVERSITETI
INTERNATIONAL BUSINESS AKADEMIK LITSEYI
O'QITUVCHISI HUSENOVA DILORAMIRZAYEVNA

dhusenova25@gmail.com

+99893 004 48 03

Abstract. Geometry is one of the oldest branches of mathematics. It developed from practical activities such as land measurement, construction, astronomy, navigation, and the study of shapes. This article examines the origin and historical development of geometry from ancient Egypt and Mesopotamia to Greek deductive geometry, Islamic scholarship, analytic geometry, non-Euclidean geometry, and modern mathematical applications. Particular attention is given to Thales, Pythagoras, Euclid, Archimedes, René Descartes, Carl Friedrich Gauss, Nikolai Lobachevsky, János Bolyai, and Bernhard Riemann. The article demonstrates how geometry evolved from empirical rules into an axiomatic and highly abstract mathematical discipline with major applications in science and technology.

Keywords: geometry, Euclid, history of mathematics, Euclidean geometry, analytic geometry, non-Euclidean geometry, axioms, geometric constructions.

1. Introduction

The word geometry comes from the Greek words *ge*, meaning earth, and *metron*, meaning measurement. The name reflects the practical origins of the subject. Early societies needed methods for measuring land, constructing buildings, determining

areas and volumes, and observing astronomical patterns. Over centuries these practical techniques were transformed into a systematic mathematical science.

Geometry studies points, lines, angles, surfaces, shapes, dimensions, positions, and spatial relationships. Its history is closely connected with the development of logical proof. Ancient Greek mathematicians did not merely calculate geometric quantities; they sought to demonstrate why geometric statements must be true.

2. Geometry in Ancient Egypt

Ancient Egyptian mathematics contained practical geometric knowledge used in surveying, architecture, agriculture, and construction. The annual flooding of the Nile created a need to re-establish land boundaries, encouraging the development of measurement techniques. Egyptian builders also required accurate methods for constructing pyramids, temples, and other structures.

Mathematical papyri show that Egyptian scribes could calculate areas of rectangles, triangles, and some other figures, as well as approximate volumes. Their geometry was primarily computational and practical rather than organized around formal axioms and proofs.

3. Geometry in Mesopotamia

Babylonian mathematicians developed sophisticated numerical and geometric methods. Clay tablets demonstrate knowledge of areas, volumes, right triangles, and numerical relationships associated with what later became known as the Pythagorean theorem. Their base-60 number system also influenced the later measurement of angles and time.

Mesopotamian geometry was closely linked with astronomy. Observations of celestial objects required systematic numerical and spatial reasoning, helping to connect geometry with the study of the heavens.

4. The Greek Transformation of Geometry

Ancient Greek mathematicians transformed geometry by emphasizing definitions, logical deduction, and proof. Instead of relying only on examples and measurements, they attempted to derive general results from accepted principles. This change was one of the most important developments in the history of mathematics.

4.1 Thales

Thales of Miletus, who lived in the sixth century BCE, is traditionally associated with some of the earliest deductive geometric reasoning in Greece. Several elementary theorems about triangles, circles, and angles were later attributed to him. His importance lies in the movement from practical measurement toward logical demonstration.

4.2 Pythagoras and the Pythagorean Tradition

Pythagoras and the mathematical school associated with his name played a major role in Greek mathematics. The Pythagorean theorem states that in a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.

$$a^2 + b^2 = c^2$$

Although knowledge of this numerical relationship existed in earlier civilizations, Greek mathematics developed rigorous approaches to proving such results. The study of irrational magnitudes also became important because the diagonal of a unit square has length $\sqrt{2}$, which cannot be expressed as a ratio of two integers.

5. Euclid and the Axiomatic Method

Around 300 BCE, Euclid of Alexandria compiled one of the most influential mathematical works in history, the *Elements*. The work organized geometric knowledge into definitions, postulates, common notions, propositions, and proofs. Its logical structure became a model for mathematical reasoning for more than two thousand years.

Euclidean geometry begins with basic concepts such as points and straight lines and derives increasingly complex theorems. Euclid's fifth postulate, commonly called the parallel postulate, later became central to the development of non-Euclidean geometry.

6. Archimedes and Geometric Measurement

Archimedes of Syracuse made major contributions to geometry, mechanics, and mathematical physics. He investigated areas and volumes of curved figures and solids, including circles, spheres, cylinders, and paraboloids. His method of exhaustion anticipated important ideas later formalized in integral calculus.

One famous result states that the volume of a sphere is two-thirds the volume of the circumscribed cylinder. Archimedes regarded his work on the sphere and cylinder as one of his greatest achievements.

7. Geometry in the Islamic Golden Age

During the medieval period, scholars working in the Islamic world translated, preserved, criticized, and extended Greek mathematical works. Euclid, Archimedes, and other classical authors became important sources for Arabic mathematical scholarship. Mathematicians studied geometric constructions, optics, astronomy, trigonometry, and the foundations of Euclidean geometry.

This scholarly tradition played an important role in transmitting mathematical knowledge to later European scholars. Geometry was also essential in architecture, astronomical instruments, mapping, and decorative geometric design.

8. The Renaissance and the Rise of Analytic Geometry

During the European Renaissance, renewed study of classical mathematics was combined with developments in art, perspective, navigation, astronomy, and engineering. The introduction of coordinate methods in the seventeenth century created a powerful connection between algebra and geometry.

René Descartes and Pierre de Fermat developed methods that allowed geometric curves to be represented by algebraic equations. In Cartesian coordinates, a point in the plane can be represented by an ordered pair (x, y) . For example, a circle with center at the origin and radius r is represented by:

$$x^2 + y^2 = r^2$$

Analytic geometry made it possible to solve geometric problems using algebra and became an essential foundation for calculus, physics, engineering, and modern mathematical modeling.

9. Projective and Differential Geometry

The study of perspective in Renaissance art helped motivate projective geometry, which investigates properties preserved under projection. Later, differential geometry used calculus to study curves and surfaces. These developments expanded geometry beyond the classical study of triangles and circles.

Carl Friedrich Gauss made fundamental contributions to the intrinsic geometry of curved surfaces. His work demonstrated that curvature can be studied from measurements made entirely within a surface, an idea that later influenced Bernhard Riemann.

10. The Birth of Non-Euclidean Geometry

For centuries mathematicians attempted to prove Euclid's parallel postulate from the other axioms. In the nineteenth century, Nikolai Lobachevsky and János Bolyai independently developed consistent geometries in which the parallel postulate was replaced by a different assumption. Their work demonstrated that Euclidean geometry is not the only logically possible geometry.

Bernhard Riemann later developed a broader theory of curved spaces. Riemannian geometry became extremely important in twentieth-century physics

because Albert Einstein's general theory of relativity describes gravity using the geometry of curved spacetime.

11. Euclidean and Non-Euclidean Geometry

Type	Main Space	Parallel Behavior	Typical Application
Euclidean geometry	Flat plane	One parallel through an external point	Classical construction and engineering
Hyperbolic geometry	Negative curvature	More than one parallel	Geometry of negatively curved spaces
Elliptic geometry	Positive curvature	No parallel lines	Spherical and curved-space models
Analytic geometry	Coordinate space	Expressed algebraically	Science, engineering and computing
Differential geometry	Smooth curves and manifolds	Depends on curvature	Relativity, mechanics and modern physics

12. Historical Timeline of Geometry

Ancient Egypt and Mesopotamia: Practical measurement, areas, volumes, surveying and astronomy.

6th century BCE: Thales and the development of deductive reasoning.

Pythagorean period: Systematic study of triangles, proportions and numerical-geometric relationships.

c. 300 BCE: Euclid organizes geometry axiomatically in the Elements.

3rd century BCE: Archimedes advances the geometry of areas and volumes.

Medieval period: Islamic scholars preserve and extend Greek geometry.

17th century: Descartes and Fermat establish analytic geometry.

19th century: Lobachevsky, Bolyai and Riemann develop non-Euclidean geometry.

20th–21st centuries: Geometry becomes fundamental to relativity, topology, computing, robotics and data science.

13. Geometry in Modern Science and Technology

Modern geometry has applications far beyond traditional measurement. Computer graphics uses coordinate and projective geometry to generate three-dimensional scenes. Robotics uses geometric transformations to describe motion and position. Geographic information systems rely on spatial geometry for mapping and navigation. Architecture and engineering use geometric modeling for structural design.

Geometry is also important in computer vision, medical imaging, aerospace engineering, crystallography, molecular modeling, machine learning, and theoretical physics. Modern fields such as computational geometry study algorithms for geometric problems, while algebraic geometry connects polynomial equations with geometric structures.

14. Educational Importance of the History of Geometry

Studying the history of geometry helps students understand that mathematical ideas developed in response to practical needs and intellectual questions. Historical examples can make formulas more meaningful because students see how concepts such as area, similarity, coordinates, and axioms emerged.

The historical development from measurement to proof also illustrates a central feature of mathematics: empirical observation may suggest a rule, but mathematical proof explains why the rule is universally valid under specified assumptions.

15. Conclusion

Geometry originated in the practical measurement activities of early civilizations and gradually developed into one of the most rigorous and influential branches of mathematics. Egyptian and Mesopotamian methods provided important practical knowledge, while Greek mathematicians transformed geometry through proof and deduction. Euclid's Elements established the axiomatic tradition, and Archimedes expanded geometric measurement. Medieval Islamic scholarship preserved and developed classical knowledge, while the Renaissance and early modern period produced analytic and projective geometry. The discovery of non-Euclidean geometry in the nineteenth century fundamentally changed the understanding of mathematical space. Today geometry remains essential in mathematics, physics, engineering, architecture, computer science, and many emerging technologies.

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