

THEME:METHODS FOR SOLVING COMBINATORIAL PROBLEMS IN MATHEMATICS

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Abstract. Combinatorics is an important branch of discrete mathematics concerned with counting, arranging, selecting, and distributing objects under specified conditions. This article presents the principal methods used to solve combinatorial problems in school and introductory university mathematics. The discussion includes the addition and multiplication principles, factorial notation, permutations, arrangements, combinations, repetition, complementary counting, and systematic case analysis. Special attention is given to identifying whether order matters and whether repetition is allowed, because these decisions determine the appropriate counting model. Worked examples demonstrate step-by-step solution strategies and common mistakes.

Keywords: combinatorics, counting principles, permutation, arrangement, combination, factorial, repetition, discrete mathematics, problem solving.

1. Introduction

Combinatorics studies finite structures and methods of counting possible configurations. It appears in probability theory, computer science, coding theory, cryptography, scheduling, optimization, and many everyday decision-making

problems. A typical combinatorial question asks how many different ways an action can be performed without listing every possibility individually.

The central difficulty is usually not computation but model selection. Before using a formula, the solver should determine what is being chosen, whether all objects or only some objects are used, whether order is important, and whether repetition is permitted.

2. Fundamental Counting Principles

2.1 Addition Principle

If one task can be completed in m ways and a mutually exclusive second task can be completed in n ways, then choosing one of the two tasks can be done in $m + n$ ways.

Total number of choices = $m + n$

Example 1. A student may choose one book from 5 mathematics books or 4 physics books. How many choices are possible?

Solution: The two categories are mutually exclusive, so $5 + 4 = 9$. Therefore, there are 9 possible choices.

2.2 Multiplication Principle

If a process consists of consecutive stages and the first stage can be performed in m ways while the second can be performed in n ways for each first-stage choice, the complete process can be performed in $m \times n$ ways.

Total number of outcomes = $m \times n$

Example 2. A student has 3 shirts and 4 pairs of trousers. How many different outfits can be formed?

Solution: For every shirt there are 4 trouser choices. Thus $3 \times 4 = 12$ outfits.

3. Factorial Notation

Factorial notation is fundamental in combinatorics. For a positive integer n , n factorial is the product of all positive integers from 1 to n . By convention, $0! = 1$.

$$n! = n(n-1)(n-2) \dots 2 \cdot 1, \quad 0! = 1$$

Example 3. Calculate $6!$.

Solution: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

4. Permutations

A permutation is an arrangement of all n distinct objects in a definite order. Because every object is used and changing the order creates a different result, the number of permutations is $n!$.

$$P_n = n!$$

Example 4. In how many ways can 5 different students stand in a row?

Solution: All five students are used and order matters. Therefore $P_5 = 5! = 120$.

Example 5. How many different arrangements can be made from the letters A, B, C and D?

Solution: There are four distinct letters, so $4! = 24$ arrangements.

5. Arrangements of k Objects from n Objects

An arrangement, also called a variation or partial permutation, selects k objects from n distinct objects when order matters. The formula is obtained by multiplying k consecutive decreasing factors beginning with n .

$$A_n^k = n! / (n - k)!$$

Example 6. From 8 students, a president and a vice-president must be selected. How many outcomes are possible?

Solution: The positions are different, so order matters. $A_8^2 = 8! / 6! = 8 \times 7 = 56$.

Example 7. How many three-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 without repetition?

Solution: Three ordered positions are filled from five digits: $A_5^3 = 5 \times 4 \times 3 = 60$.

6. Combinations

A combination selects k objects from n distinct objects when order does not matter. Choosing students A, B and C produces the same group as choosing C, A and B. Therefore the permutations within each selected group must be divided out.

$$C_n^k = n! / [k!(n - k)!]$$

Example 8. How many committees of 3 students can be selected from 10 students?

Solution: Order is irrelevant. $C_{10}^3 = 10!/(3!7!) = (10 \times 9 \times 8)/(3 \times 2 \times 1) = 120$.

Example 9. A class has 7 students. How many different pairs can be formed?

Solution: A pair is an unordered selection, so $C_7^2 = 7!/(2!5!) = 21$.

7. Permutations with Repeated Objects

If some objects are identical, ordinary factorial counting overcounts equivalent arrangements. If n objects contain groups of $n_1, n_2, \dots, n_r$ identical objects, the number of distinguishable permutations is:

$$P = n! / (n_1! n_2! \dots n_r!)$$

Example 10. How many distinct arrangements are possible for the letters of the word LEVEL?

Solution: There are 5 letters, with L repeated twice and E repeated twice. Thus $5!/(2!2!) = 30$.

8. Counting with Repetition

When repetition is allowed, the number of choices can change substantially. If each of k ordered positions can independently contain any of n symbols, then there are n^k sequences.

Number of ordered sequences with repetition = n^k

Example 11. How many four-digit PIN codes can be formed using digits 0–9 if repetition is allowed?

Solution: Each of the four positions has 10 choices. Therefore $10^4 = 10,000$ PIN codes.

9. Complementary Counting

Sometimes it is easier to count all possible outcomes and subtract the outcomes that violate the required condition. This method is called complementary counting.

Required outcomes = All outcomes – Unwanted outcomes

Example 12. How many three-digit numbers contain at least one digit 7?

Solution: There are 900 three-digit numbers in total. Without digit 7, the hundreds digit has 8 choices (1–9 except 7), while each remaining digit has 9 choices. Thus $8 \times 9 \times 9 = 648$ numbers contain no 7. Therefore $900 - 648 = 252$ numbers contain at least one 7.

10. Systematic Case Analysis

A complicated problem can often be divided into several non-overlapping cases. The number of outcomes in each case is calculated separately and then combined using the addition principle. This method is especially useful when restrictions affect different positions in different ways.

Example 13. How many two-digit even numbers can be formed from 1, 2, 3, 4 and 5 without repetition?

Solution: The units digit must be 2 or 4, giving 2 choices. For each choice, the tens digit can be any of the remaining 4 digits. Therefore $2 \times 4 = 8$.

11. How to Choose the Correct Method

Situation	Are all objects used?	Does order matter?	Main method
Arrange all distinct objects	Yes	Yes	Permutation: $n!$
Select k objects for ranked positions	No	Yes	Arrangement: A_n^k
Select a group of k objects	No	No	Combination: C_n^k
Independent consecutive choices	Depends	Usually yes	Multiplication principle
Mutually exclusive alternatives	Depends	No	Addition principle
Count 'at least one' efficiently	Depends	Depends	Complementary counting

12. A Practical Problem-Solving Algorithm

A reliable combinatorial solution can be organized into five steps. First, identify the objects and the required outcome. Second, decide whether the process is a selection or an arrangement. Third, determine whether order matters. Fourth, check whether repetition is allowed and whether there are additional restrictions. Finally, choose the appropriate counting principle or formula and verify that no outcome has been counted twice.

A useful decision rule is: if all objects are arranged, use a permutation; if only k of n objects are chosen and order matters, use an arrangement; if only k of n objects are chosen and order does not matter, use a combination.

13. Common Errors

The most common mistake is using combinations when order matters or using arrangements when order does not matter. Another frequent error is ignoring restrictions such as repeated digits, leading zeros, identical objects, or mandatory selections. Students should translate verbal conditions into precise mathematical choices before performing calculations.

It is also important to avoid double counting. If the same final group can be produced through several different orders, those orders should not be counted as different outcomes unless the problem explicitly distinguishes them.

14. Applications of Combinatorics

Combinatorial methods are used in probability calculations, password analysis, network design, tournament scheduling, database queries, genetics, coding theory, algorithm analysis, and artificial intelligence. For example, combinations determine the number of possible groups, while permutations model rankings and sequences. In probability, counting favorable outcomes and total outcomes is often the first step in calculating a probability.

15. Conclusion

Combinatorial problem solving is based on understanding the structure of a counting situation. The addition and multiplication principles provide the foundation, while permutations, arrangements, and combinations offer efficient formulas for common selection and ordering problems. Repetition, restrictions, and complementary counting extend these methods to more complex situations. The most important skill is not memorizing formulas but correctly identifying whether order matters, whether

all objects are used, and whether repetition is permitted. A systematic approach makes combinatorial problems clearer, faster, and less prone to error.

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