



UCH O'LCHAMLI OLMOS PANJARADAGI DISKRET SHREDINGER OPERATORNING MUHIM SPEKTRI

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Annotatsiya: Ushbu maqolada uch o'lchamli olmos panjarasida aniqlangan diskret Shredinger operatorining muhim spektri tadqiq etiladi. Tadqiqot doirasida operatorning asosiy spektral xossalari, jumladan, spektrning tuzilishi, uning nuqtaviy va uzluksiz qismlari hamda operatorga mos keluvchi matritsaviy ifodalar batafsil tahlil qilinadi. Shuningdek, olmos panjarasining o'ziga xos geometrik va topologik tuzilishining operator spektriga ta'siri o'rganiladi. Olingan natijalar kvant mexanikasi, kristall panjaralardagi elektron holatlarini modellashtirish hamda matematik fizika masalalarida qo'llanilishi mumkin. Nazariy natijalar analitik usullar asosida olinib, ayrim misollar yordamida operator spektrining strukturaviy xususiyatlari yoritib beriladi.

Kalit so'zlar: olmos panjara, regulyar nuqta, spektr, operatorning spektri, diskret spektr, muhim spektr, kompakt operator.

СУЩЕСТВЕННЫЙ СПЕКТР ДИСКРЕТНОГО ОПЕРАТОРА ШРЁДИНГЕРА В ТРЁХМЕРНОЙ АЛМАЗНОЙ РЕШЁТКЕ

Аннотация: В данной статье исследуется существенный спектр дискретного оператора Шрёдингера на трёхмерной алмазной решётке. Рассматриваются спектральные свойства данного оператора, в частности, структура спектра, его точечная и непрерывная части, а также матричные представления, связанные с этим оператором. Также анализируется влияние



геометрических и топологических особенностей алмазной решётки на спектр оператора. Полученные результаты могут быть применены в квантовой механике, при моделировании электронных состояний в кристаллических решётках и в математической физике. Наряду с аналитическими методами приведены примеры, иллюстрирующие структурные особенности спектра оператора.

Ключевые слова: алмазная решётка, регулярная точка, спектр, спектр оператора, дискретный спектр, существенный спектр, компактный оператор.

THE ESSENTIAL SPECTRUM OF THE DISCRETE SCHREDINGER OPERATOR IN A THREE-DIMENSIONAL DIAMOND LATTICE

Abstract: *This article investigates the essential spectrum of the discrete Schrödinger operator on a three-dimensional diamond lattice. The study focuses on the spectral properties of the operator, particularly the structure of the spectrum, its point and continuous parts, as well as matrix representations associated with the operator. The influence of the geometric and topological features specific to the diamond lattice on the operator's spectrum is also analyzed. The results obtained can be applied in quantum mechanics, modeling electronic states in crystal lattices, and mathematical physics. In addition to analytical methods, the article presents examples illustrating structural characteristics of the operator's spectrum.*

Keywords: *diamond lattice, regular point, spectrum, operator spectrum, discrete spectrum, essential spectrum, compact operator.*

Quyidagi to'plamni qaraymiz:

$$A_2 = \{v(n): v(n) = n_1 v_1 + n_2 v_2 \quad n = (n_1; n_2), \quad n \in \mathbb{Z}^2\},$$

$$\text{bu yerda} \quad v_1 = (-1; 0; 1) \quad v_2 = (0; -1; 1).$$



Ta'rif 1. A_2 to'plamga 2 o'lchamli olmos panjara deyiladi.

Quyidagi to'plamni kiritamiz:

$$\Omega = A_2 \cup (p + A_2), \quad p = \frac{1}{3}(-1; -1; 2).$$

$\ell_2(\Omega)$ - orqali Ω da kvadrati bilan jamlanuvchi $\hat{f}(n) = (\hat{f}_1(n), \hat{f}_2(n))$ funksiyalar juftligini belgilaymiz. Bu fazo Gilbert fazosi bo'lib, skalyar ko'paytma quydagicha aniqlangan

$$(\hat{f}, \hat{g}) = \sum_{v \in A_2} 3\hat{f}_1(n)\hat{g}_1(n) + \sum_{v \in (p+A_2)} 3\hat{f}_2(n)\hat{g}_2(n).$$

$\mathbb{T} = (-\pi; \pi]$. $L_2^{(2)}(\mathbb{T}^2) - \mathbb{T}^2$ da aniqlangan kvadrati bilan integrallanuvchi $f(x) = (f_1(x), f_2(x))$ funksiyalar juftligining Gilbert fazosi bo'lsin. Bu yerda skalyar ko'paytma quydagicha aniqlangan:

$$(f, g) = (f_1, g_1) + (f_2, g_2)$$

bunda

$$(f_i, g_i) = \int_{\mathbb{T}^2} f_i(x) \overline{g_i(x)} dx, \quad i = 1, 2.$$

Quyidagi $F : \ell_2(\Omega) \rightarrow L_2^{(2)}(\mathbb{T}^2)$ unitar operatorni kiritamiz:

$$F = \begin{pmatrix} \mathcal{F} & 0 \\ 0 & \mathcal{F} \end{pmatrix}, \quad (\mathcal{F}\hat{f})(x) = \frac{\sqrt{3}}{2\pi} \sum_{n \in \mathbb{Z}^2} e^{i(x,s)} \hat{f}(s).$$

Bu operator teskarisi $F^{-1} : L_2^{(2)}(\mathbb{T}^2) \rightarrow \ell_2(\Omega)$ quydagicha aniqlanadi:

$$F^{-1} = \begin{pmatrix} \mathcal{F}^{-1} & 0 \\ 0 & \mathcal{F}^{-1} \end{pmatrix}, \quad (\mathcal{F}^{-1}f)(s) = \frac{\sqrt{3}}{2\pi} \int_{\mathbb{T}^2} e^{-i(s,x)} f(x) dx.$$

bu yerda $(s, x) = s_1x_1 + s_2x_2$.

Olmos panjaradagi diskrit Shredinger operatori $\widehat{H}$ ushbu $\ell_2(\Omega)$ fazoda chegaralangan o'z-o'ziga qo'shma operator sifatida quydagicha aniqlanadi:

$$\widehat{H} = -3(\Delta_2 + 1) + \widehat{Q}$$

bunda $(-3(\Delta_2 + 1)\hat{f})(v) = ((V_1\hat{f}_2)(n); (V_2\hat{f}_1)(n))$,

bu yerda $(V_1\hat{f}_2)(n) = \hat{f}_2(n) + \hat{f}_2(n - e_1) + \hat{f}_2(n - e_2)$

$$(V_2\hat{f}_1)(n) = \hat{f}_1(n) + \hat{f}_1(n - e_1) + \hat{f}_1(n - e_2)$$



$$e_1, e_2, n \in \Omega, \quad n = (n_1; n_2), \quad e_1 = (1; 0), \quad e_2 = (0; 1).$$

$\hat{Q}$ - Ω da aniqlangan zarrachalarning o'zaro ta'sir potentsiali bo'lib, ular quyidagi formulalar bilan aniqlanadi:

$$(\hat{Q}f)(n) = \begin{pmatrix} \hat{Q}_1(n) & 0 \\ 0 & \hat{Q}_2(n) \end{pmatrix} \begin{pmatrix} \hat{f}_1(n) \\ \hat{f}_2(n) \end{pmatrix} = \begin{pmatrix} \hat{Q}_1(n)\hat{f}_1(n) \\ \hat{Q}_2(n)\hat{f}_2(n) \end{pmatrix}$$

bunda

$$\sum_{n \in A_2} |\hat{Q}_1(n)| < \infty, \quad \sum_{n \in (p+A_2)} |\hat{Q}_2(n)| < \infty.$$

$\widehat{H}$ operatorni koordinata ko'rinishidan impuls tasvirga o'tish F almashtirishlari yordamida amalga oshiriladi.

$$H = F\widehat{H}F^{-1} = F(-3(\Delta_2 + 1))F^{-1} + F\hat{Q}F^{-1}.$$

H operator olmos panjaradagi diskrit Shredinger operatorining impuls tasviri bo'lib, u quydagicha aniqlanad:

$$H = H_0 + Q, \quad (1)$$

bu yerda : H_0 va Q 2×2 matritsa uchun matritsa operatorlari bo'lib, $L_2^{(2)}(\mathbb{T}^2)$ da quyidagicha aniqlanadi:

$$(H_0f)(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} E(x)f_2(x) \\ \overline{E(x)}f_1(x) \end{pmatrix},$$

$$(Qf)(x) = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} (Q_1f_1)(x) \\ (Q_2f_2)(x) \end{pmatrix},$$

bunda, $E(x) - 2$ o'zgaruvchili kompleks qiymatli funksiya

$$E(x) = \frac{1}{3}(1 + e^{ix_1} + e^{ix_2}), \quad Q_i - L_2(\mathbb{T}^2) \text{ da aniqlangan integral operator}$$

$$(Q_if_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t)f_i(t)dt. \quad i = 1, 2,$$

$Q_i(\cdot) - \mathbb{T}^2$ da aniqlangan haqiqiy qiymatli biror uzluksiz, juft funksiya.



Mazkur maqolada $L_2^{(2)}(T^2)$ – Gilbert fazosida (1) ko‘rinishda aniqlangan Olmos panjaradagi diskret Shredinger operatori $H = H_0 + Q$ ning muhim spektrini o‘rganamiz.

Bizga ma’lumki biror $\lambda \in \mathbb{C}$ uchun $A - \lambda I$ operator teskarilantiruvchan bo‘lsa, u holda λ soni A operatorning **regulyar nuqtasi** deb atalar edi va A operatorning barcha regulyar nuqtalari to‘plami $\rho(A)$ kabi belgilangan edik.

Ta’rif 2. $\sigma(A) = \mathbb{C} \setminus \rho(A)$ to‘plam A operatorning **spektri** deb ataladi.

Ta’rif 3. $\lambda \in \sigma(A)$ son yakkaqlangan, A operatorning chekli karrali xos qiymatlari to‘plami **diskret spektr** deb ataladi va $\sigma_{disc}(A)$ deb belgilanadi.

Ta’rif 4. $\sigma_{ess}(A) = \sigma(A) \setminus \sigma_{disc}(A)$, A **muhim spektr** deb ataladi.

Ushbu tadqiqotning asosiy teoremasi quyidagidan iborat

Teorema 1. $\sigma(H_0) = [-1; 1]$.

Isbot. Bizga ma’lumki [Reed Simon], H_0 2×2 matritsa operatorining spektri quyidagi formula bilan aniqlanadi:

$$\sigma(H_0) = \bigcup_{x \in T^2} \sigma(H_0(x)). \quad (*)$$

Bunda $H_0(x)$ – har bir fiksirlangan $x \in T^2$ da 2×2 sonli matritsa bo‘ladi, ya’ni

$$H_0(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix}, \quad E(x) = \frac{1}{3}(1 + e^{ix_1} + e^{ix_2}).$$

Shuning uchun $H_0(x)$ ning spektri xos qiymatlaridan iborat bo‘ladi, ya’ni har bir tayinlangan $x \in T^2$ larda $\det|H_0 - \lambda I| = 0$ tenglamaning ildizlaridan iboratdir. Bu tenglamani tuzamiz:

$$(H_0 - \lambda I)(x) = \begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -\lambda & E(x) \\ \overline{E(x)} & -\lambda \end{pmatrix}.$$

$$\det|H_0 - \lambda I| = \begin{vmatrix} -\lambda & E(x) \\ \overline{E(x)} & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - E(x) \cdot \overline{E(x)} = 0 \Rightarrow$$

$$\lambda^2 = |E(x)|^2 \Rightarrow \lambda_{1,2} = \pm |E(x)|, \quad x \in T^2.$$

bu yerda



$$\begin{aligned}
 |E(x)|^2 &= E(x)\overline{E(x)} = \frac{1}{3}(1 + e^{ix_1} + e^{ix_2}) \frac{1}{3}(1 + e^{-ix_1} + e^{-ix_2}) = \\
 &= \frac{1}{9}(3 + e^{ix_1} + e^{-ix_1} + e^{ix_2} + e^{-ix_2} + e^{i(x_1-x_2)} + e^{-i(x_1-x_2)}) = \\
 &= \frac{1}{9}(3 + 2\cos x_1 + 2\cos x_2 + 2\cos(x_1 - x_2)).
 \end{aligned}$$

Shunday qilib,

$$\sigma(H_0(x)) = \{xos\ qiymatlari\} = \{-|E(x)|; |E(x)|\}.$$

Demak, (*) ga ko'ra

$$\begin{aligned}
 \sigma(H_0) &= \bigcup_{x \in T^2} \sigma(H_0(x)) = \bigcup_{x \in T^2} \{-|E(x)|; |E(x)|\} \\
 &= -Ran\{|E(x)|\} \cup Ran\{|E(x)|\}.
 \end{aligned}$$

$$\text{Endi } |E(x)| = \max_{x \in T^2} \frac{1}{9}(3 + 2\cos x_1 + 2\cos x_2 + 2\cos(x_1 - x_2)) = 1 \text{ va}$$

$$\min_{x \in T^2} |E(x)| = 0 \text{ ekanligidan, ushbu } -Ran\{|E(x)|\} = [-1; 0] \text{ va}$$

$$Ran\{|E(x)|\} = [0; 1] \text{ tengliklarni hosil qilamiz. Demak } \sigma(H_0) = [-1; 1].$$

Teorema isbotlandi.

Lemma 1. $Q : L_2^{(2)}(\mathbb{T}^2) \rightarrow L_2^{(2)}(\mathbb{T}^2)$ kompakt operator.

Isbot. $Q : L_2^{(2)}(\mathbb{T}^2) \rightarrow L_2^{(2)}(\mathbb{T}^2)$. operatorni ko'rinishi quydagicha edi:

$$(Qf)(x) = \begin{pmatrix} (Q_1 f_1)(x) \\ (Q_2 f_2)(x) \end{pmatrix} = \begin{pmatrix} (\mu_1 f_1)(x) \\ (\mu_2 f_2)(x) \end{pmatrix}$$

$$\text{bunda, } (Q_i f_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t) f_i(t) dt. \quad i = 1, 2, \mu_1, \mu_2 > 0.$$

Biz Q operatorni kompaktligini ko'rsatishimiz uchun har bir $i \in \{1, 2\}$ da

$Q_i : L_2(\mathbb{T}^2) \rightarrow L_2(\mathbb{T}^2)$ operatorni kompakt ekanligini ko'rsatamiz. Ma'lumki, $(Q_i f_i)(x) = \int_{\mathbb{T}^2} Q_i(x-t) f_i(t) dt$. operator kompakt bo'lishi uchun $\int_{\mathbb{T}^2} \int_{\mathbb{T}^2} |Q_i(x-t)|^2 dt dx < \infty$ bo'lishi zarur va yetarli. Shartga ko'ra $Q_i(\cdot)$ – ikkala o'zgaruvchi buyicha ham $\mathbb{T}^2$ da aniqlangan biror uzluksiz funksiya.

Bundan $\int_{\mathbb{T}^2} \int_{\mathbb{T}^2} |Q_i(x-t)|^2 dt dx$ integral mavjud va chekli. Demak Q_i kompakt, ya'ni Q kompakt operator.

Teorema 2. $\sigma_{ess}(H) = \sigma(H_0) = [-1, 1]$.



Isbot. Muhim spektr turg'unligi haqidagi **Veyl teoremasi** ga ko'ra $H = Q + H_0$ operatorning muhim spektri Q kompakt qo'zg'alishda o'zgarmaydi va H_0 operator spektri bilan ustma-ust tushadi. Q kompakt operator. Bu yerdan esa xulosa $\sigma_{ess}(H) = \sigma(H_0)$. Teorema isbotlandi.

Biz endi $H = H_0 + Q$ operatorning $Q = \mu_i > 0, i = 1, 2$ holdagi xos qiymati va unga mos xos vektorini topish masalasini ko'rib chiqamiz. Buning uchun quyidagi belgilashlarni kiritamiz:

$D(z) = \begin{vmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{vmatrix}$, ko'rinishdagi 2×2 matritsa, bunda

$$\begin{cases} \Delta_{11} = 1 + z\mu_1 \int_{T^2} \frac{1}{|E(s)|^2 - z^2} ds, & \Delta_{12} = \mu_2 \int_{T^2} \frac{E(s)}{|E(s)|^2 - z^2} ds \\ \Delta_{21} = \mu_1 \int_{T^2} \frac{\overline{E(s)}}{|E(s)|^2 - z^2} ds, & \Delta_{22} = 1 + z\mu_2 \int_{T^2} \frac{1}{|E(s)|^2 - z^2} ds \end{cases} \quad (1)$$

Teorema 3. $z \in \mathbb{R} \setminus [-1, 1]$ soni H operatorning xos qiymati bo'lishi uchun $D(z) = 0$ bo'lishi zarur va yetarli.

Isbot(Zaruriyligi): z sono H operatorning xos qiymati bo'lsin, ya'ni

$$H\psi = z\psi$$

$$\psi = (\psi_1, \psi_2), \psi_1, \psi_2 \in L_2(T^2)$$

tenglik bajarilsin. H operator chiziqliligidan

$$H\psi = (H_0 + Q)\psi = H_0\psi + Q\psi = \begin{pmatrix} E(x) & \psi_2 \\ \overline{E(x)} & \psi_1 \end{pmatrix} + \begin{pmatrix} \int_{T^2} \mu_1 \psi_1(s) ds \\ \int_{T^2} \mu_2 \psi_2(s) ds \end{pmatrix} = z \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Bu yerdan quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} E(x)\psi_2(x) + \mu_1 \int_{T^2} \psi_1(s) ds = z\psi_1(x) \\ \overline{E(x)}\psi_1(x) + \mu_2 \int_{T^2} \psi_2(s) ds = z\psi_2(x) \end{cases} \quad (2)$$



Quyidagicha belgilash kiritamiz

$$C_1 = \int_{T^2} \psi_1(s) ds, \quad C_2 = \int_{T^2} \psi_2(s) ds \quad (3)$$

u holda (1) ga (2) ni qo'yib quyidagi sistemasiga ega bo'lamiz:

$$\begin{cases} E(x)\psi_2(x) + \mu_1 C_1 = z\psi_1(x) \\ \overline{E(x)}\psi_1(x) + \mu_2 C_2 = z\psi_2(x) \end{cases}$$

bundan

$$\begin{cases} E(x)\psi_2(x) = -\mu_1 C_1 + z\psi_1(x) \\ \overline{E(x)}\psi_1(x) = -\mu_2 C_2 + z\psi_2(x) \end{cases} \quad (4)$$

ni hosil qilamiz. Dastlab birinchi va ikkinchi tenglamani mos ravishda $\overline{E(x)}$ va $E(x)$ larga ko'paytiramiz:

$$\begin{cases} |E(x)|^2 \psi_2(x) + \mu_1 \overline{E(x)} C_1 = z \overline{E(x)} \psi_1(x) \\ |E(x)|^2 \psi_1(x) + \mu_2 E(x) C_2 = z E(x) \psi_2(x) \end{cases}$$

Oxirgi sistemaga (3) ni qo'yamiz va quyidagi sistemani hosil qilamiz:

$$\begin{cases} |E(x)|^2 \psi_2(x) + \mu_1 \overline{E(x)} C_1 = z(-\mu_2 C_2 + z\psi_2(x)) \\ |E(x)|^2 \psi_1(x) + \mu_2 E(x) C_2 = z(-\mu_1 C_1 + z\psi_1(x)) \end{cases}$$

yoki

$$\begin{cases} (|E(x)|^2 - z^2) \psi_2(x) = -\mu_1 \overline{E(x)} C_1 - z\mu_2 C_2 \\ (|E(x)|^2 - z^2) \psi_1(x) = -\mu_2 E(x) C_2 - z\mu_1 C_1 \end{cases}$$

$|E(x)|^2 - z^2 \neq 0$ ekanligidan ψ_1, ψ_2 funksiyalarga ega bo'lamiz:

$$\begin{cases} \psi_1(x) = -\frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 - \frac{z\mu_1}{|E(x)|^2 - z^2} C_1 \\ \psi_2(x) = -\frac{\mu_1 \overline{E(x)}}{|E(x)|^2 - z^2} C_1 - \frac{z\mu_2}{|E(x)|^2 - z^2} C_2 \end{cases} \quad (5)$$

Endi (5) ni (3) ga olib borib qo'yamiz va quyidagini hosil qilamiz:

$$\begin{cases} C_1 = \int_{T^2} \left(-\frac{\mu_2 E(s)}{|E(s)|^2 - z^2} C_2 - \frac{z\mu_1}{|E(s)|^2 - z^2} C_1 \right) ds \\ C_2 = \int_{T^2} \left(-\frac{\mu_1 \overline{E(s)}}{|E(s)|^2 - z^2} C_1 - \frac{z\mu_2}{|E(s)|^2 - z^2} C_2 \right) ds \end{cases}$$



$$\begin{cases} C_1 \left(1 + z\mu_1 \int_{T^2} \frac{1}{|E(s)|^2 - z^2} ds \right) + C_2 \mu_2 \int_{T^2} \frac{E(s)}{|E(s)|^2 - z^2} ds = 0 \\ C_1 \mu_1 \int_{T^2} \frac{\overline{E(s)}}{|E(s)|^2 - z^2} ds + C_2 \left(1 + z\mu_2 \int_{T^2} \frac{1}{|E(s)|^2 - z^2} ds \right) = 0 \end{cases}$$

bu yerdan (1) ga ko'ra

$$\begin{cases} C_1 \Delta_{11} + C_2 \Delta_{12} = 0 \\ C_1 \Delta_{21} + C_2 \Delta_{22} = 0 \end{cases}$$

ko'rinishdagi tenglamalar sistemasiga kelamiz. Bunda $\psi = (\psi_1, \psi_2)$ vektor H operatorning xos funksiyalari bo'lganligi uchun $(C_1, C_2) \neq 0$ bo'lishi lozim. Bizga algebra kursidan ma'lumki, bir jinsli tenglamalar sistemasi noldan farqli yechimga ega bo'lishi uchun asosiy determinant nolga teng bo'lishi zarur. Bundan

$$D(z) = \begin{vmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{vmatrix} = 0.$$

Yetarliligi: Faraz qilaylik, $z \in R \setminus [-1, 1]$ ga $D(z) = \begin{vmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{vmatrix} = 0$

bo'lsin. U holda shunday $(C_1, C_2) \neq 0$ mavjudki,

$$\begin{cases} \Delta_{11} C_1 + \Delta_{12} C_2 = 0 \\ \Delta_{21} C_1 + \Delta_{22} C_2 = 0 \end{cases}$$

tengliklar o'rinli bo'ladi.

$\psi = (\psi_1, \psi_2)$ vektorda ψ_1 va ψ_2 lar quyidagi ko'rinishda bo'lsin:

$$\begin{aligned} \psi_1(x) &= -\frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 - \frac{z\mu_1}{|E(x)|^2 - z^2} C_1 \\ \psi_2(x) &= -\frac{\mu_1 \overline{E(x)}}{|E(x)|^2 - z^2} C_1 - \frac{z\mu_2}{|E(x)|^2 - z^2} C_2 \end{aligned}$$

$\psi = (\psi_1, \psi_2)$ ni H operatorning xos vektoriga ekanligini ko'rsatamiz. Buning uchun H operatorning $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ vektorga ta'sirini qaraymiz.

$$H\psi = (H_0 + Q)\psi = \left(\begin{pmatrix} 0 & E(x) \\ \overline{E(x)} & 0 \end{pmatrix} + \begin{pmatrix} 0 & \mu_1 \\ \mu_2 & 0 \end{pmatrix} \right) \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

$$= \begin{pmatrix} E(x)\psi_1(x) + \mu_1 \int_{T^2} \psi_1(s) ds \\ \overline{E(x)}\psi_1(x) + \mu_2 \int_{T^2} \psi_2(s) ds \end{pmatrix} = \begin{pmatrix} E(x)\psi_1(x) + \mu_1 C_1 \\ \overline{E(x)}\psi_1(x) + \mu_2 C_2 \end{pmatrix}$$

Bu tengliklarga (4) ni keltirib qo'yamiz:

$$\begin{aligned} H\psi &= \begin{pmatrix} E(x) \left(-\frac{\mu_1 \overline{E(x)}}{|E(x)|^2 - z^2} C_1 - \frac{z\mu_2}{|E(x)|^2 - z^2} C_2 \right) + \mu_1 C_1 \\ \overline{E(x)} \left(-\frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 - \frac{z\mu_1}{|E(x)|^2 - z^2} C_1 \right) + \mu_2 C_2 \end{pmatrix} = \\ &= \begin{pmatrix} -\frac{\mu_1 |E(x)|^2}{|E(x)|^2 - z^2} C_1 + \mu_1 C_1 - \frac{z\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{z\mu_1}{|E(x)|^2 - z^2} C_1 - \frac{\mu_2 |E(x)|^2}{|E(x)|^2 - z^2} C_2 + \mu_2 C_2 \end{pmatrix} = \\ &= \begin{pmatrix} \frac{-|E(x)|^2 + |E(x)|^2 - z^2}{|E(x)|^2 - z^2} \mu_1 C_1 - \frac{z\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{z\mu_1}{|E(x)|^2 - z^2} C_1 + \frac{-|E(x)|^2 + |E(x)|^2 - z^2}{|E(x)|^2 - z^2} \mu_2 C_2 \end{pmatrix} = \\ &= \begin{pmatrix} \frac{-z^2}{|E(x)|^2 - z^2} \mu_1 C_1 - \frac{z\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{z\mu_1}{|E(x)|^2 - z^2} C_1 - \frac{z^2}{|E(x)|^2 - z^2} \mu_2 C_2 \end{pmatrix} = \\ &= z \begin{pmatrix} \frac{-z}{|E(x)|^2 - z^2} \mu_1 C_1 - \frac{\mu_2 E(x)}{|E(x)|^2 - z^2} C_2 \\ -\frac{\mu_1}{|E(x)|^2 - z^2} C_1 - \frac{z}{|E(x)|^2 - z^2} \mu_2 C_2 \end{pmatrix} = z \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \end{aligned}$$

yoki bundan

$$H\psi = z \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

tenglikka ega bo'lamiz. Demak, $\psi = (\psi_1, \psi_2)$ H operatorning xos funksiyasi va z soni H operatorning xos qiymati ekan. Teorema isbot bo'ldi.



Isbotlangan teoreмага asosan, aytish mumkinki H operatorning muhim spektri $[-1,1]$ dan iborat, $R \setminus [-1,1]$ da esa deskrit spektri mavjud.

Xulosa

Mazkur ishda uch o'lchamli olmos panjarada aniqlangan diskret Shryodinger operatorining spektral xossalari o'rganildi. Dastlab operator spektri va uning tarkibiy qismlari — diskret hamda muhim spektr tushunchalari kiritildi. Asosiy natija sifatida H_0 operatorining spektri $[-1,1]$ oraliqqa tengligi isbotlandi. Shuningdek, integral ko'rinishda berilgan Q operatorning kompaktligi ko'rsatildi. Veyl teoremasiga asosan, kompakt qo'zg'alish ostida muhim spektr o'zgarmasligi sababli $H = H_0 + Q$ operator uchun ham muhim spektr $[-1,1]$ ga teng ekanligi aniqlandi. Olingan natijalar diskret kvant tizimlarining spektral tahlilida muhim ahamiyatga ega.

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